Fluid Mechanics
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Lecture - 20
Dimension Analysis and Similarity

Welcome all of you for this lectures on fluid mechanics and today we will discuss about dimension analysis and the similarity.

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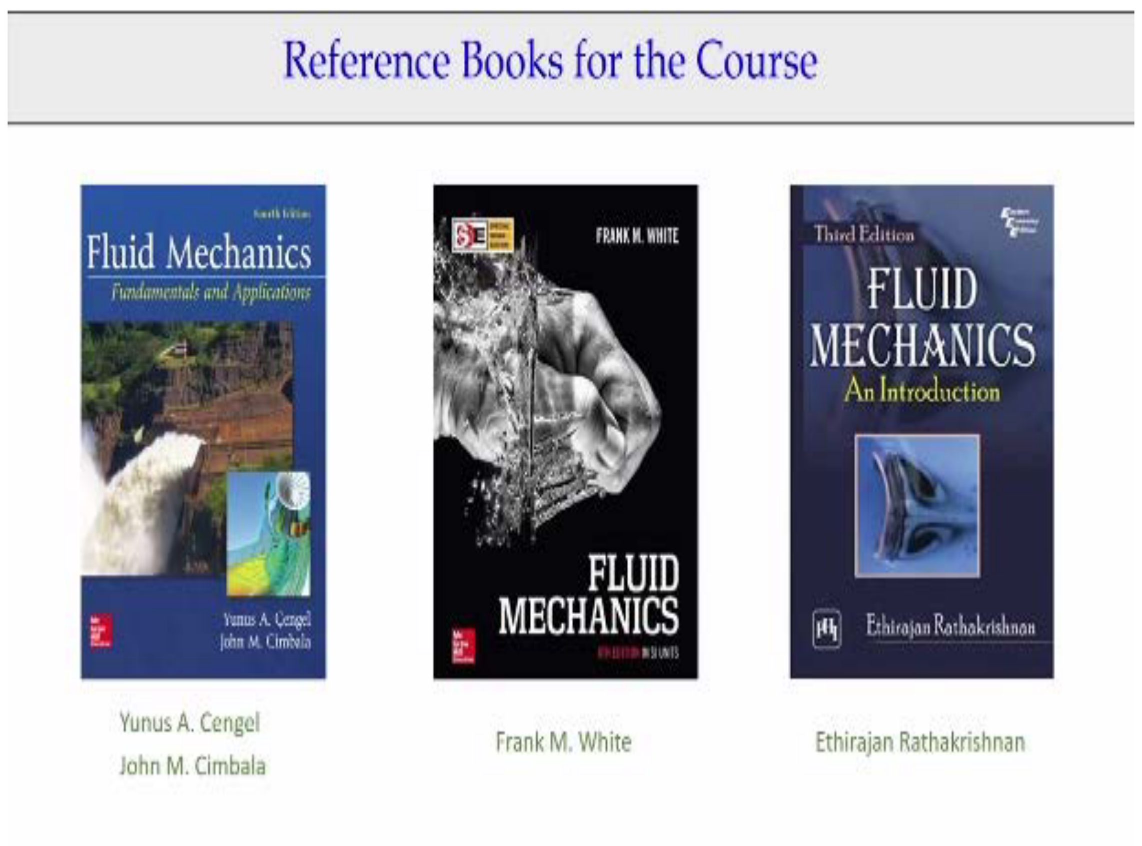

I have been following these 3 reference books, but the mostly today I will talk about the professor Radhakrishnan's books, which is one of the (()) (00:53) on the dimension analysis and the similarity concept.

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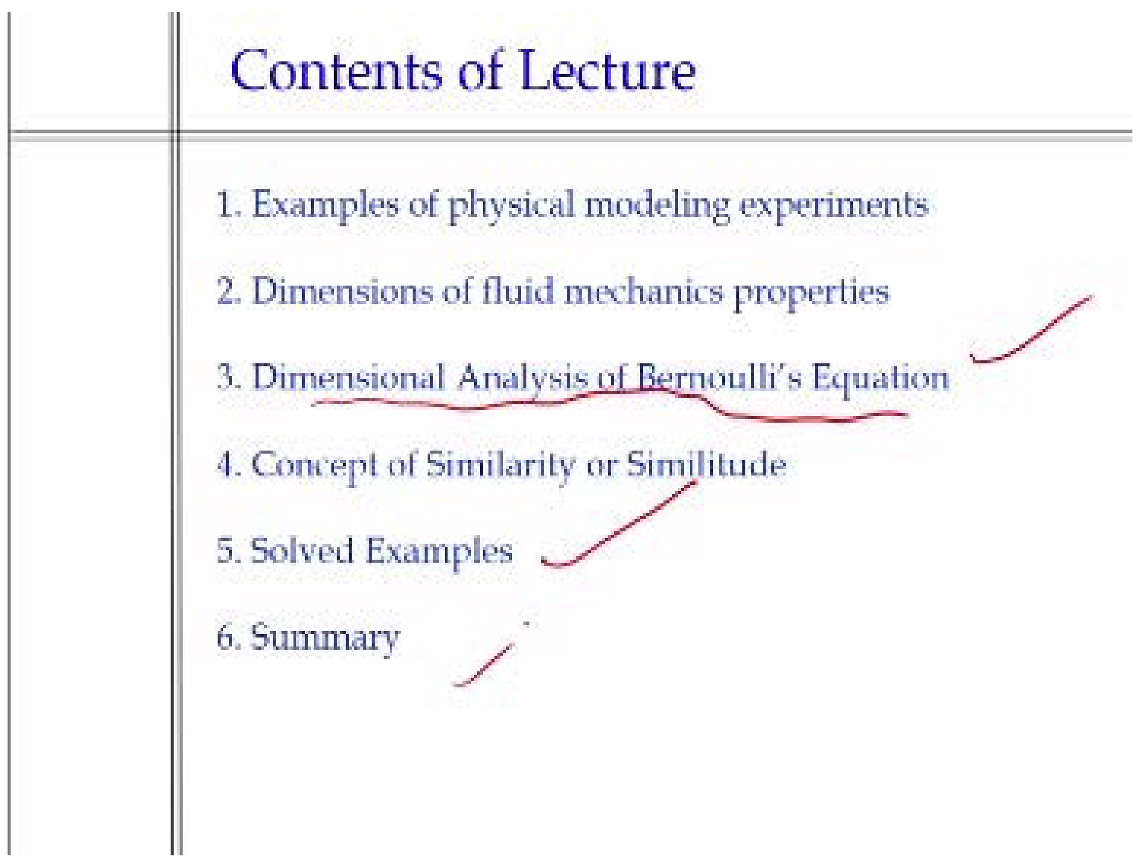

Now if you talk about that I will start with the physical modelling experiments, what we have in IIT Guwahati or elsewhere, how we conduct the physical modelling experiment. Then I will talk about the dimensions of fluid mechanics properties which last class we discussed, but just I have to repeat it and more interesting things today I will talk about the dimensional analysis of Bernoulli's equation okay.

Then I will talk about the concept of similarity or similitude, solved examples and summary. Let us look at very interesting study.

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What we have been doing in the previous lab in IIT Guwahati. So one sites of the figures if you look at this Brahmaputra river, it is big river systems which have the dimensions in terms of the length of the island 6 kilometre, width of the island 2.5 kilometre. So huge dimensions in terms of the length and the width of the islands, the discharge is about 10,000 metre cube per second, huge discharge, the particle size.

Then width of the left channel and the right channel, these are the dimensions of real rivers and we call the dimensions of prototype. So the real river dimensions what is that is given as a prototype dimensions, that is what we reflect in the modelling setups what we have, where we scale down it with the distance level as well as the geometry level we scale down it, like for example, the length of the island at the prototype level is 6.5 kilometres.

But the dimensions what you have put it here is about 1.73 metres. So that is what is called geometric similarity. So we scaled down the models with an appropriate scale. Similar way also we have scaled down the discharge which is at the prototype level it is 10,000 metre cube per second, but at the models or at the prove levels we tested with a discharge of 10 litre per seconds.

So how to do this type of calculations what could be the model discharge so that it can represent the flow behaviours at the prototype levels. This is a biggest question what today will be answered and if you look at these 2 figures you can see these are the 2 experimental data or experimental geometries of this river represented for the Brahmaputra rivers.

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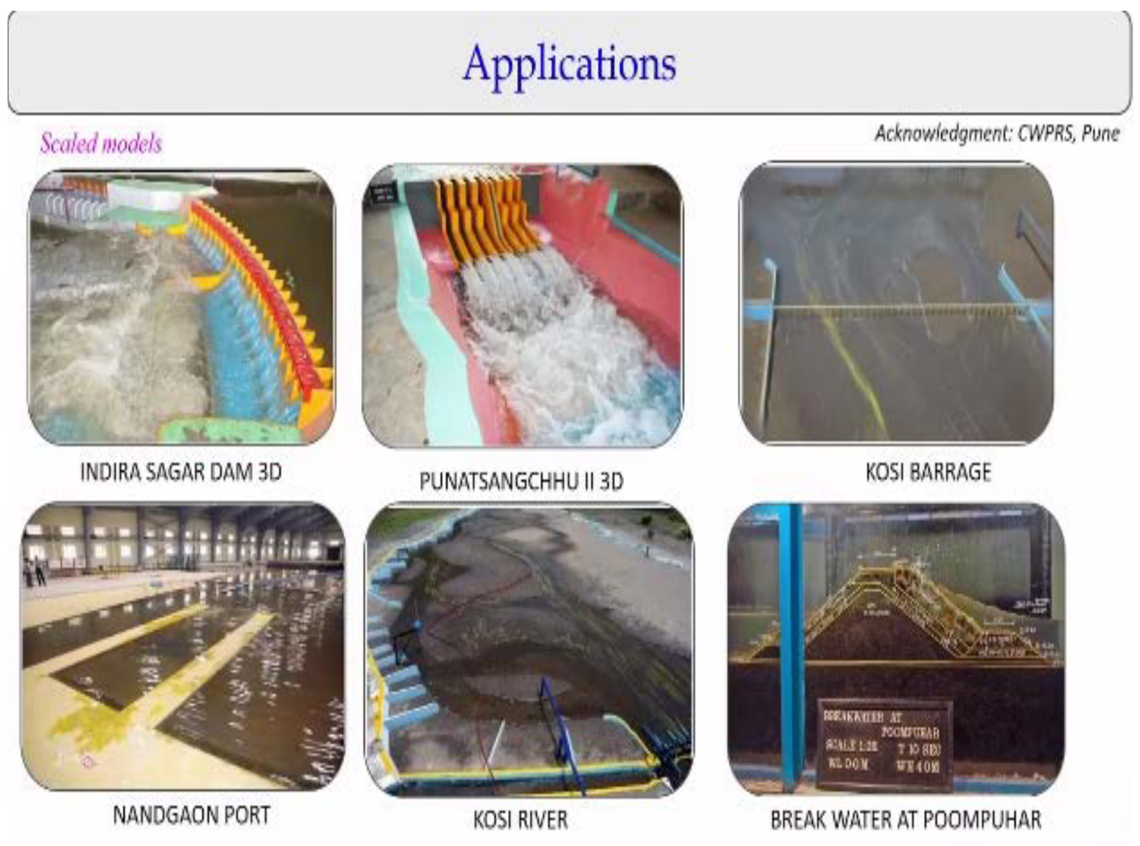

Same way if you look at that there are series of physical models conducted in CWPRS, Pune. So like the Kosi barrage if you know it the Kosi barrage, this is the physical models of the Kosi barrage, which is the scale down the physical model of Kosi barrage. Similar way Indira Sagar Dam 3D models if you can look it which is a scale down models here. Same way the Nandaon port, Kosi river barrage, break water at Poompuhar and 3D models of this ones.

If you look at these models there are the scale down models from the prototypes, but when you do these also we try to look other similarities, what are they? That is what today we will discuss it, but these type of experimental set up we conduct before implementing major projects like barrage, the dam projects, the port projects, all the major project before implementing it these all we tested.

And tried to find out what could be the optimum solutions for these Kosi barrage as well as the Kosi river projects. Like for example, if you can see it there are the flow lines are there, you can see these flow lines. You can see here there is energy dissipation is happening, you just see it, where you can see it, if you conduct a physical model you can really just observe the flow patterns.

You can see this flow patterns, how the flow is going on. It is very easy to observe the flow pattern, the stream lines, the vertex formations all we can see it very closely conducting this experiment. Similar way if you are looking it how much of energy dissipated just downstream of the spin wheel or the fluids gates you can easily find out how these, the eddies and the vortex, the energy dissipations what is happening.

And after that how the hydraulic jump or missions are happening, all these things we can study when you conduct series of physical models. Most of the civil engineering projects before implementing the field, we conduct a series of physical modelling test also we conduct numerical models what available today in different commercially or free softwares we also conduct the experiment.

But I can say that most of the times we relay, we depend upon the physical model because that what gives a lot of the strength the support to engineer to take the decision because they can see visually how the flow (()) (07:21) happening it, like for example, of the Kosi barrage, if you look into the flow patterns, all the things we can visualize it with a different flow conditions.

So consider this visualisations part considering the flow patterns to observe even if in a scale down models that is what gives a lot of confidence to the engineers before implementing. So I can say there is a 2 complementing each other. The computational fluid dynamics and the scale models they are complimenting each others. We cannot construct a barrage or the dam without conducting physical models.

So both are complementing each other in terms of understanding what will be the flow (()) (08:13) what will be the energy dissipation all we can understand conducting series of physical models. Now you can have the questions in your mind then apart from the geometric similarity what you can see it always a scale models have a scale down model and sometimes also use a scale of models that means always tell it a 1:10 or 1:100, the scale down models.

That means 1 unit of the models representing 10 unit of prototypes or 1 unit of models representing 100 units of in the prototypes. So we do the scale down models to represent the things also sometimes we also go for scale of models. The models are bigger size than the prototype, but these are very special scales. So of you look at this way we have a lot of applications of the physical models conducted or any major river projects or the dam projects.

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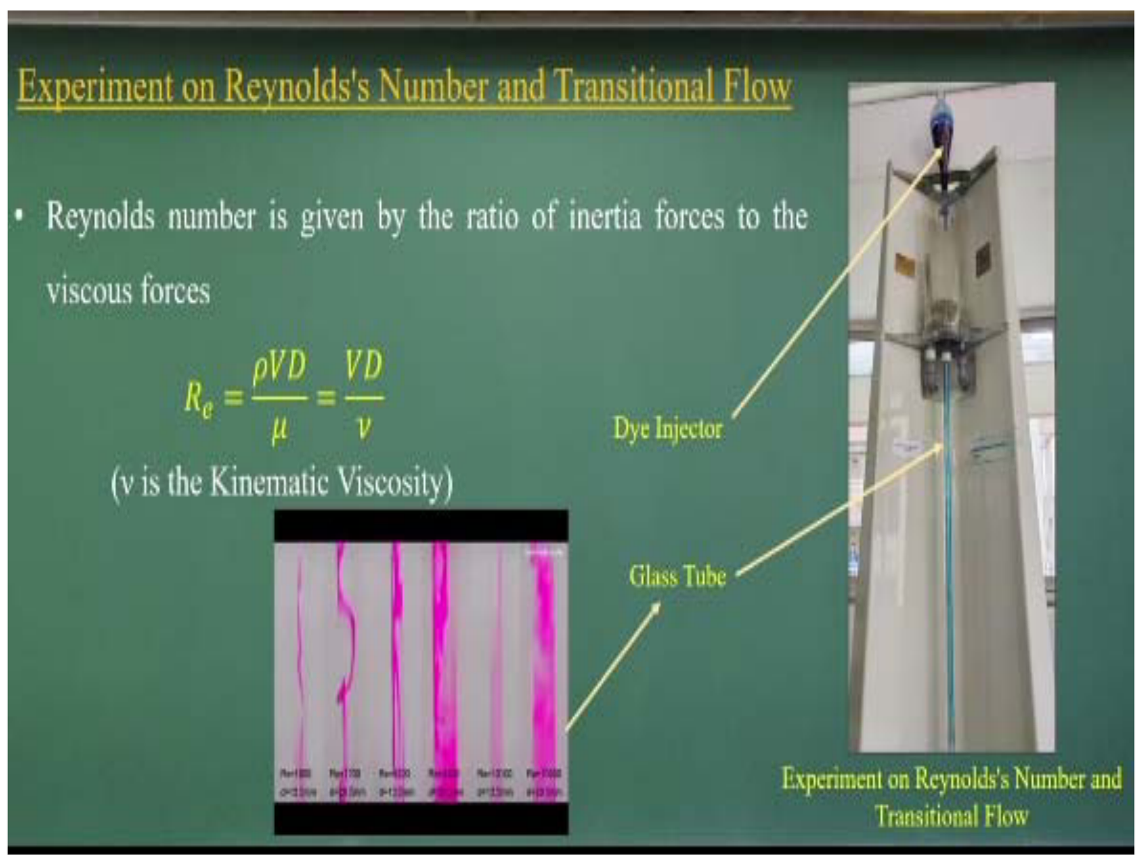

Now if you look it before starting the topic on similarities, I can say that let us have do a small experiments which is called Reynolds number apparatus experiment which is very simple experiment what was conducted by Reynolds with a colour dye facilities, is a dye injectors and there is a pipe which is regulated the floor here and the flow can be conducted with different Reynolds numbers, the flow Reynolds numbers.

The flow Reynolds number as you know it is defined as this way,

$$R_e = \frac{\rho VD}{\mu} = \frac{VD}{\nu}$$

that means inertia force by viscous force, if I make it that receives will gives Reynolds number, but this that what as you increase the number of the Reynolds numbers as is indicated this that, if you increase the number of Reynolds number you can see this coloured dyes are changing it, that means there are lot of diffusions of coloured dyes happening.

The flow patterns are changing it, visually what we are saying it, there is change of the flow patterns, there is change of flow from the laminar to transitions to the turbulent. We will discuss all how it happens with virtual fluid ball concepts but you can see that with simple coloured dye experiments we can find out at what the Reynolds numbers, the flow changes from lamina to transitions.

The transitions stage to the turbulent stage, we can identify and we conduct the experiment with respect to the nondimensional number of Reynolds numbers. So that way this is a very simple experiment to find out the threshold of the Reynolds numbers to divide between laminar transitional flow, transitional flow and the turbulent flow. So more details we will discuss in the next class.

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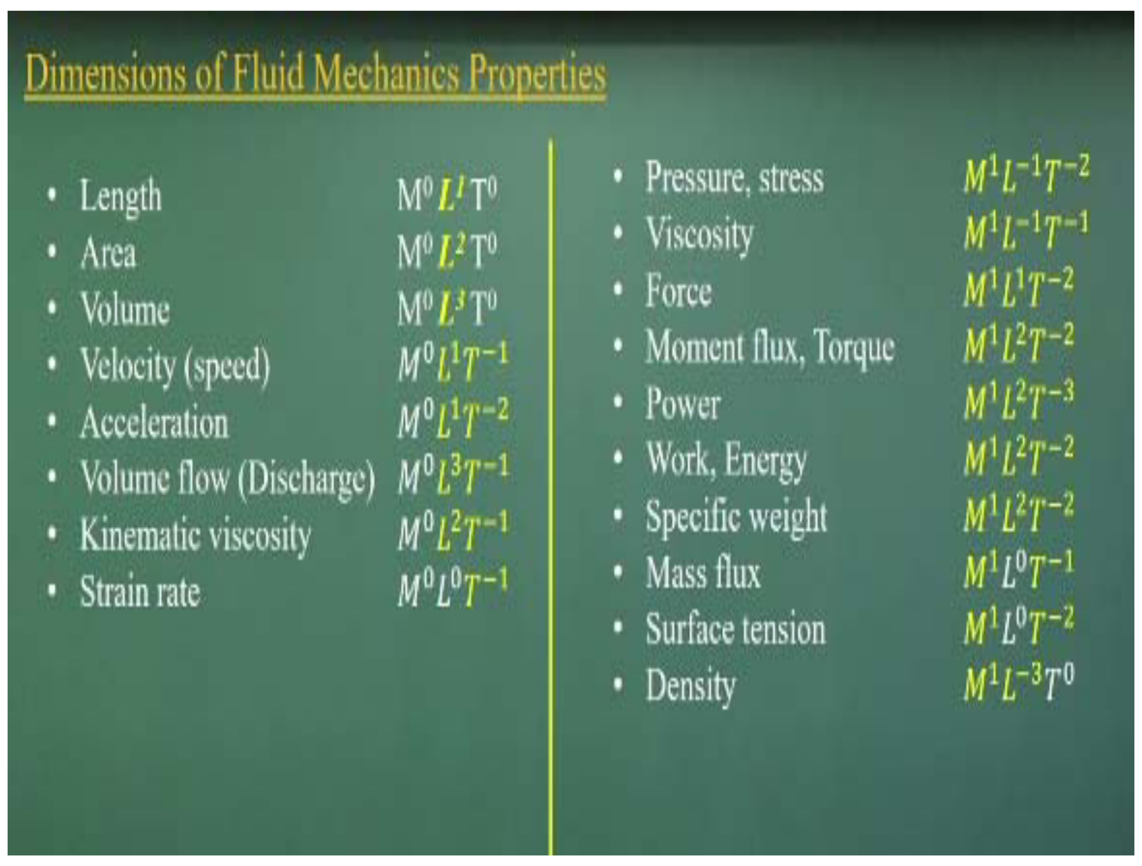

Now let us coming to the dimensions of the fluid mechanics properties, as discussed in the last class, I am just repeating it that you have the dimensions of basic geometry which in terms of length, area, volume. Then comes out the velocity, accelerations and volumetric discharge that means volume per unit time, that is what the discharge, volumetric discharge, volumetric rate.

Kinematic viscosity and the strain rate, similar way we look it this sides, the dimensions of a pressure, stress, viscosity, momentum flux and torque, power, work and energy, mass flux, this is the momentum flux, this is for the mass flux surface tension and the density. So I am not repeating these, but just to have the, showing you the dimensions of basic fluid properties, what we generally encounter to solve the problems.

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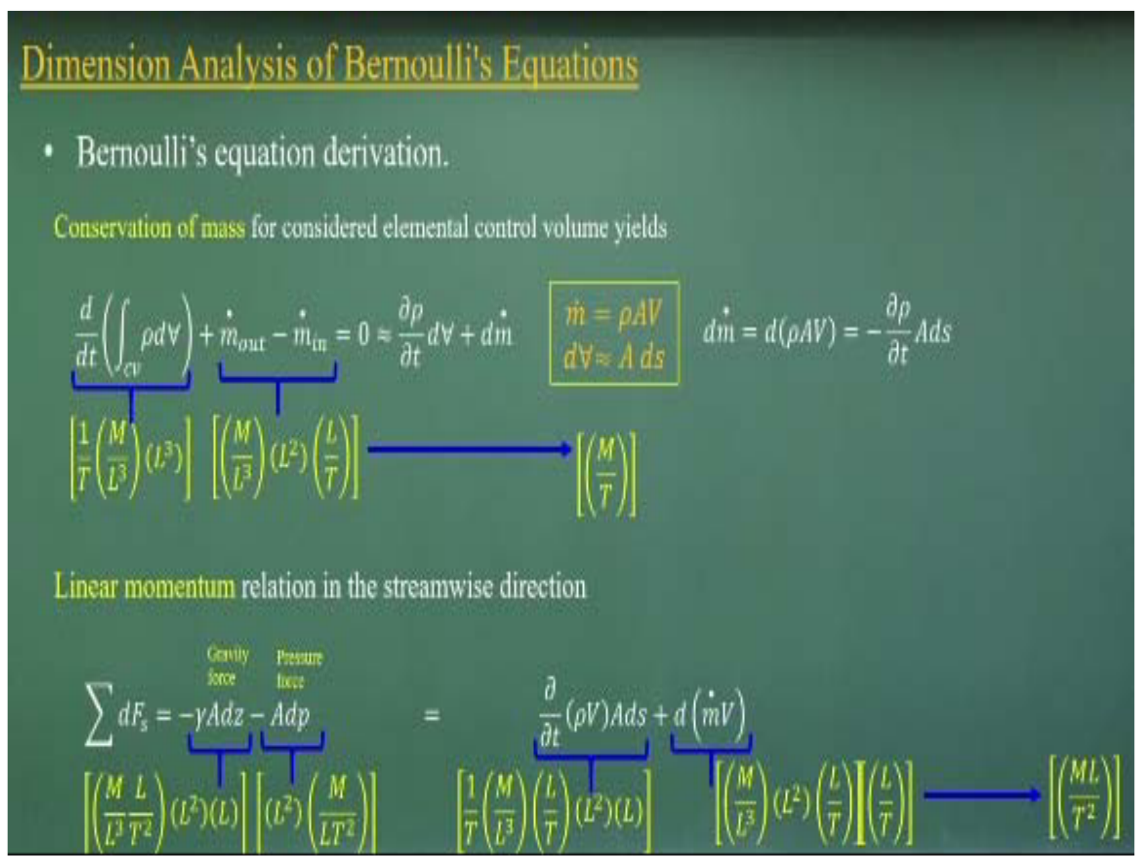

Now if you look it, many of the times when we derive a big equations, we face a difficulty or we face have a doubt over that, the equations whatever I have derive it are they correct? In a equation you can have soft components, are the soft components are correct? So what do we do it, we use a dimensional analysis concept to find out the equations what we have derived is it correct, also we can look at the equation derivation at the step by step that what is the unit.

$$\frac{d}{dt}\left(\int_{cv}\rho d\forall\right)+\dot{m}_{out}-\dot{m}_{in}=0\approx\frac{\partial\rho}{\partial t}d\forall+d\dot{m}$$

$$d\dot{m}=d(\rho AV)=-\frac{\partial\rho}{\partial t}Ads$$

$$\frac{d}{dt}\left(\int_{cv}\rho d\forall\right)+\dot{m}_{out}-\dot{m}_{in}=0\approx\frac{\partial\rho}{\partial t}d\forall+d\dot{m}$$

$$\left[\frac{1}{T}\left(\frac{M}{L^3}\right)(L^3)\right]\left[\left(\frac{M}{L^3}\right)(L^2)\left(\frac{L}{T}\right)\right]\rightarrow\left[\left(\frac{M}{T}\right)\right]$$

There is a equations which is involved with volumetricals and divide it there is a mass flux in and mass flux out, it looks like very difficult equations. It is having that stands that we will do mistakes in any of the part of the equations.

What you do it that case, that you just check the correctness of this equations, substitute the dimensions of each terms like $\rho d\forall$, d∀ is the volume stands for, $\frac{1}{T}$ is the time, $\frac{d}{dt}$ is $\frac{1}{T}$. So you get dimensions of these part. Similar way this is mass flux coming in and going out, that is what VV and by T. So that way if you look it the each terms of this equations having the same dimensions that means the equation is correct.

The same way you can try it look this ones, this also will have a same expressions. So we can always look it whenever you have a doubt, over a equations which is mostly the lengthy equations you substitute the dimensions and see that each term of the equations whether have the same dimension or not. If not, then you check it where it is the mistakes. So that is what in this case, the conservation of mass which look is very complicated equations.

But to know the correctness of the equations we use the dimensions of each term then find out what is the dimension of each term then the part what we look it at the dimensionally homogeneous, that is what we tested it. Similar way this if you look at the linear momentum relation in the streamwise direction which looks very difficult equations what we have, we write it from these, the sum of force is gravity force and pressure force, again we write in this form.

$$\sum dF_s = -\gamma A dz - A dp \qquad = \qquad \frac{\partial}{\partial t}(\rho V) A ds + d\left(\dot{m} V\right)$$

$$\left[\left(\frac{M}{L^3}\frac{L}{T^2}\right)(L^2)(L)\right]\left[(L^2)\left(\frac{M}{LT^2}\right)\right] \qquad \left[\frac{1}{T}\left(\frac{M}{L^3}\right)\left(\frac{L}{T}\right)(L^2)(L)\right]\left[\left(\frac{M}{L^3}\right)(L^2)\left(\frac{L}{T}\right)\right] \rightarrow \left[\left(\frac{ML}{T^2}\right)\right]$$

It you look it there is a differences, so all these terms if I substitute the one by one the dimensions like in this case gravity force, this is the unit weight, area, then dz, this is a area dp, the pressures. Similar way if I substitute all the terms if you look at that each terms having dimensions into L by T square which is the force. So each terms are equating into the force.

So these concept what I introduce for you only whenever you derive a big equations, complicated equations like there will be a integral part, differences part, please use the dimensional analysis compound to find out are they correct in terms of concept of dimensional homogenous of this equation. Most of the physical equations are dimensionally homogeneous.
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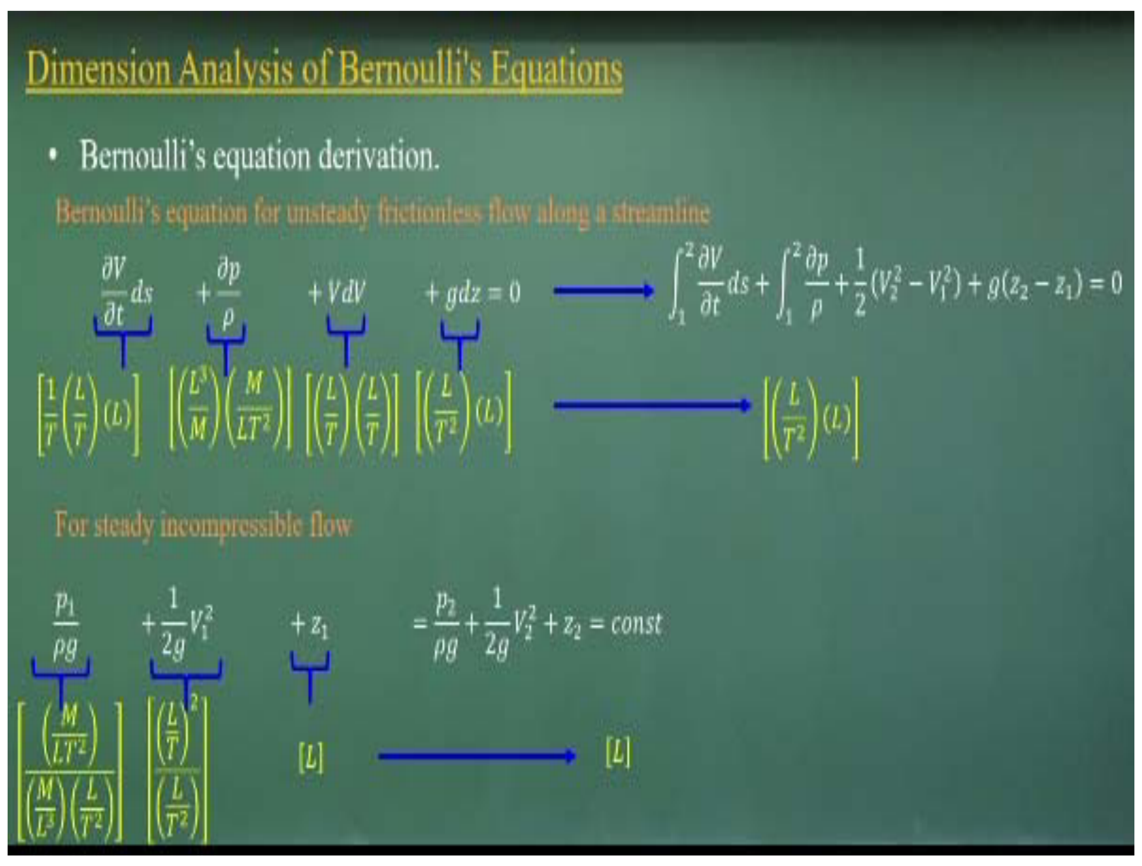

Now the similar way you know it is most often what we do in the fluid mechanics, we use the Bernoulli's equations, okay, mostly widely used or widely misused equations, which is called the Bernoulli's equation. We discussed lot about this equations. So let us have again look at that in a dimensional analysis of Bernoulli's equations with a derivations part. If you look at the unsteady frictionless flow along a streamline which is the Bernoulli's equations, it look it in terms of differences, the integrations part and all the things substitute the dimensions.

$$\frac{\partial V}{\partial t}ds \qquad + \frac{\partial p}{\rho} \qquad + VdV \qquad + gdz = 0$$

Finally, we will see it the each terms having the same dimensions $\left[\left(\frac{L}{T^2}\right)(L)\right]$. Substitute that, I just encouraged the students have these practise, wherever you derive the equations, just spends few minutes to write the dimensions of these variables, each term, then check it whether it is dimensional homogenous. So that way you can use each dimensions and check it this is what.

The similar way you can check this equations also. If there is a mistake in this equations you can easily identify when you equate the dimension of each terms, because it is dimensional homogeneous equations. So each term should have the same dimensions and if you equate it you will get same thing. So if you go for a simple model this is what called a very, it is not a complex that merge, but we can say that having integrations and all the fact.

So but when you talk about a steady and compress flow, you know the standard Bernoulli's equation in a different forms and you just substitute it, you will get the dimensions which is the length dimensions. So we always use the dimensions to understand it whether is there is a correctness of the equations, if not then we look it where is the mistakes. Not only that you can also interpret it each term what is representing it.

For steady incompressible flow

$$\frac{p_1}{\rho g} + \frac{1}{2g}V_1^2 + z_1 = \frac{p_2}{\rho g} + \frac{1}{2g}V_2^2 + z_2 = const$$

$$\left[\frac{\left(\frac{M}{LT^2}\right)}{\left(\frac{M}{L^3}\right)\left(\frac{L}{T^2}\right)}\right] \quad \left[\frac{\left(\frac{L}{T}\right)^2}{\left(\frac{L}{T^2}\right)}\right] \quad [L] \quad \rightarrow \quad [L]$$

Like for example, what I have here is the length, that means it is an edge equivalent to a water head. It is a length dimension, it is L equal to energy in terms of water head, that is what we are getting it here. So the dimensions also speaks out that what type of conservations we are doing it, but we can also interpret it the physical meaning of this dimensions what we are getting it.

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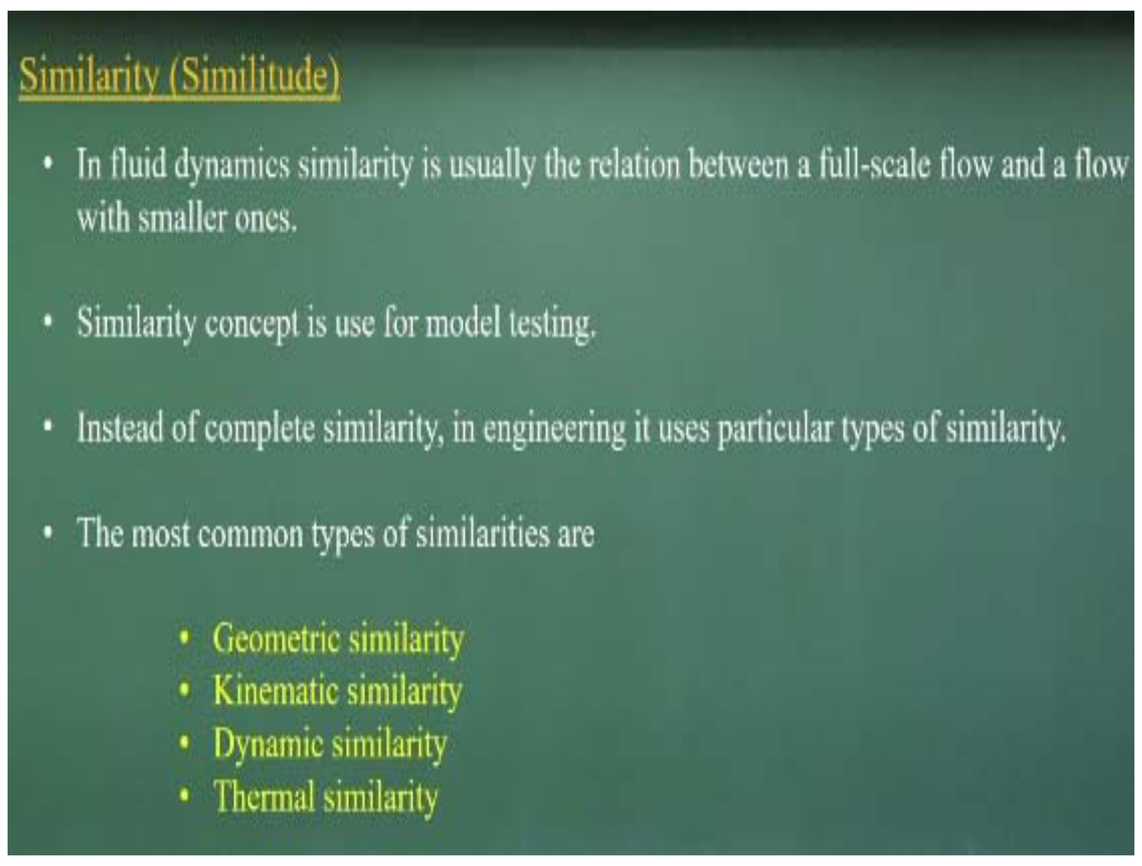

So looking that part, let us go for the next step which is the similarity or similitude (()) (20:13) that means in fluid dynamics when you conduct the experiments, we should have a relationship between full scale or the prototype and the flow with smaller ones which is the model. We need to have a relationship between that. That relationship if you can use it by conducting the experiment at the model scale you can take it to the prototype scale.

If that is the conditions either you conduct the experiment with the full scale as we saw lot of experiment facilities for automobile industry they do, to tell full scale models okay, but many of the cases we cannot go for the full scale models like a dam which is in generally of having the dimensions for 30 meters high, it is width of 40 meters, more than that, we cannot conduct these type of big experiment scale experiment in any big set up.

So we cannot do a full scale models for a dam, full scale model for the barrage, but we can conduct full scale models for automobile aerospace industry problems also, but not in the problems which we encounter in civil engineering problems, which have more dimensions we scaled down the models which we do the scale down of the models we have to look the similarity.

This 4 type of the similarities happens geometric similarity, kinematic similarity, dynamic similarity and the thermal similarity. So as we are not talking about energy, conservations much more in these lectures, we focus on these 3 similarity, geometry, kinematics and the dynamic similarity.

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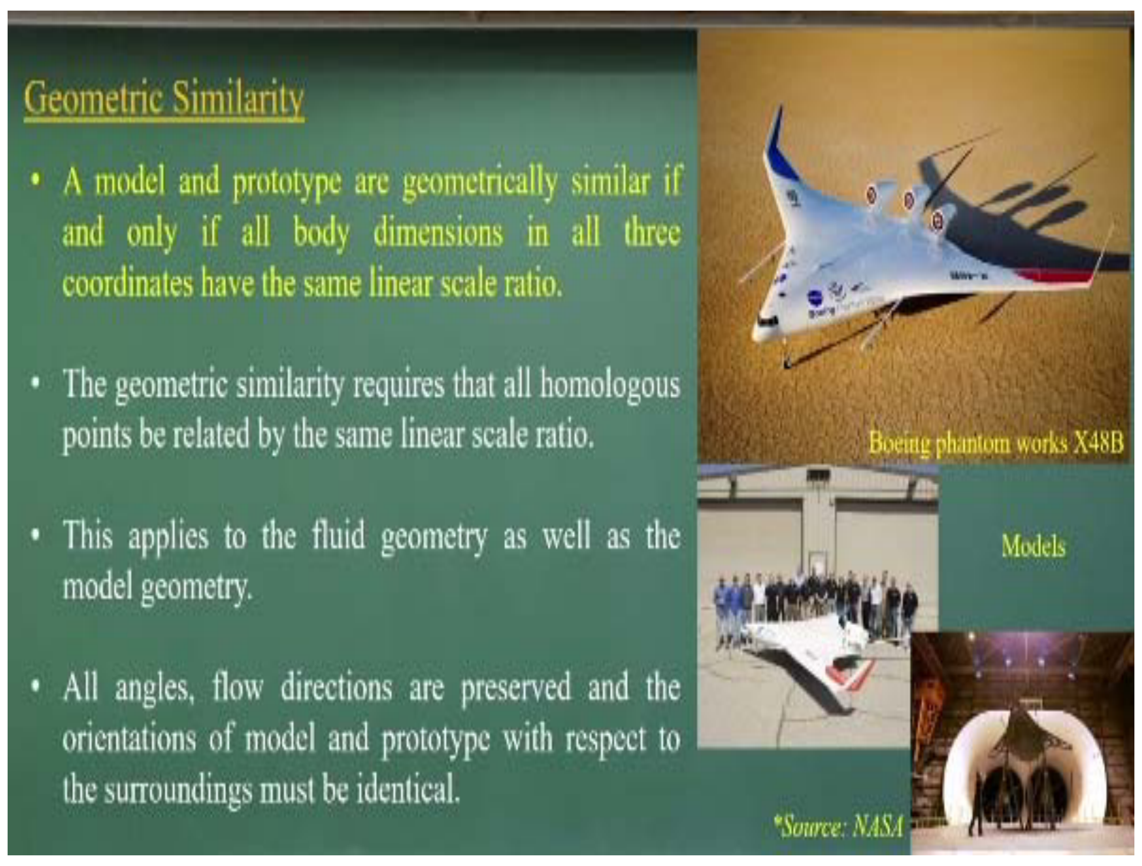

So if we look in that part, the first I am talking about geometric similarity. If you look it, this is what the prototypes, pole scale okay, that is what is the Boeing X48B but we cannot conduct this big scale experiments, what we do it we scale down it. We make it exactly the same in terms of a scale down models so that we keep all the angle, the flow direction are preserved.

Orientations models and prototype with respect to surrounding must be identical. So what we do it we do a geometrical similarity with a same linear scale ratio. Same liner scale ratio we apply it and make it to geometrically the similarity that is what is the geometrical and similarity models what exactly you can compare visually that they are geometrical scale down it.

They are like the length, the area, the volume, all geometric scale down with the angles and all this and finally, we testing in the wind tunnel facility, that is what the wind tunnel facilities this model is tested in the wind tunnel facility and whatever the pressure variations, the velocity variations, they we take it from the model to the prototype to design a robust Boeing fighter plane. So this is what have been doing it with the geometric similarity.

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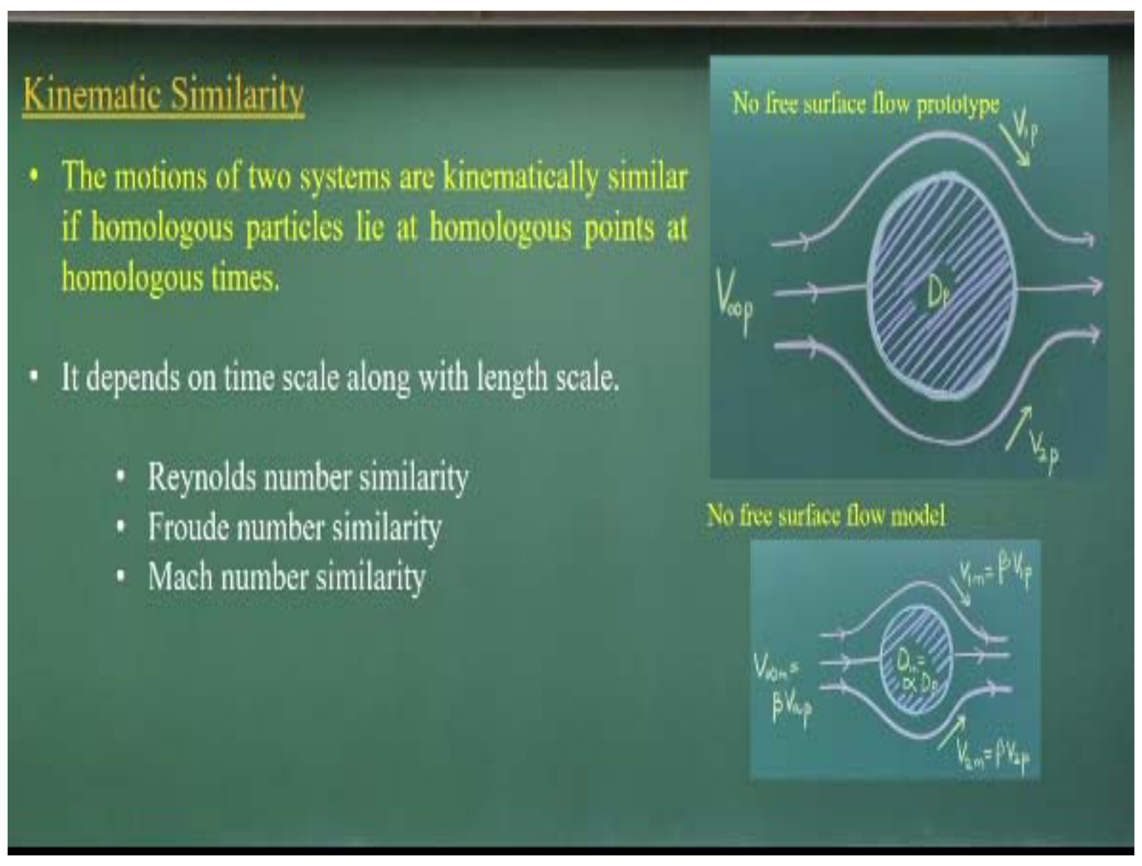

But we also to look it should have a kinematic similarity. When you talk about the kinematics what it says that we have the model, we have the prototypes, that this is the prototypes, this is the models, that means in the model and prototypes would have the same velocity factor, the stream lines patterns that means the velocity factors what we have the velocity direction that stream line and the models would have a scale factors okay.

We can have a scale factor so that the kinematic similar is a homologous points, the points what we have to telling it which is the representing models and the prototypes that should have a lies at the homologous points at the homologous times and same time in time and also in a positions, this would have the same location, same factors, so we will represent it. If it is that then it is called the kinematics similarities.

So it depends of the time scale along with the length scale. So 2 scales are there. The time scale and the length scale. So to achieve this we do a Reynolds number similarity, Froude number similarity or Mach number similarity.

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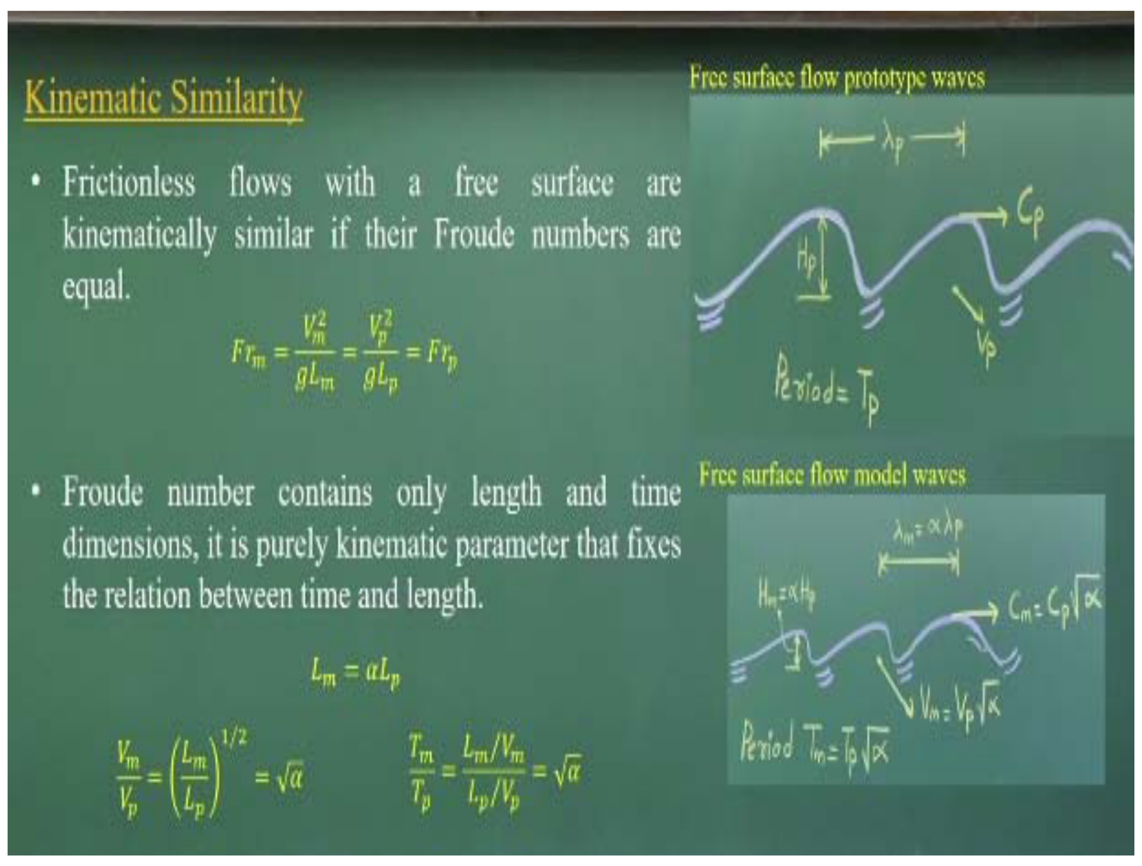

Now if you look at another problem where we are looking for the similarity length, like for example, we are looking for a free surface flow with a prototype waves. This is the wave process maybe happening in oceans or the lake that what we have to represent it in model waves. When you do that if you look at that the way we have characteristics like the height of the wave, the length of the wave and also we have the velocity factors.

In these case since it is a free surface flow conditions, flow Froude numbers should be equal or model as well as for the prototype, that is what is there, the flow Froude numbers of the model and prototype should be equal then we are substituting the flow Froude numbers in terms of the velocity and the length.

$$Fr_m = \frac{V_m^2}{gL_m} = \frac{V_p^2}{gL_p} = Fr_p$$

Since we are conducting the experiment on the R surface so G does not vary, the acceleration due to gravity does not vary.

So you can use the same G components, so you can get it the a relationship between the velocity at the model, length velocity of the model and the velocity and the length of the prototypes there is a relationship what we get it when you do kinematic similarity, that means equating flow Froude numbers or model is equal to flow Froude number of the prototype.

So if you look at the scales, so the flow Froude numbers contains the length and the time dimensions because there is no mass dimensions in that. So it is purely kinematic parameters fixes the relation between length and time scale. If I consider the alpha scaling factors between the prototypes and the models if I use this relationship, I can get the what could be the ratio of the velocity of the models and the velocity of prototypes with substituting this equations will be square root of the alpha.

$$L_m = \alpha L_p$$

$$\frac{V_m}{V_p} = \left(\frac{L_m}{L_p}\right)^{1/2} = \sqrt{\alpha}$$

Similar way the time scale will be computed as like this, so these are very simple calculations can be done. So if you look at that way, this is what the prototype scale after the scaling down the same process with scale down the length in terms of alpha factors which is the length of the wave and the height of the waves but the velocity and the time will have a different scale factors. This is what we computed based on flow Froude number similarity concept.

$$\frac{T_m}{T_p} = \frac{L_m/V_m}{L_p/V_p} = \sqrt{\alpha}$$

So this is what the time ratio or time scale and the velocity scale as make it, so most of the times we do these things to compute it what could be the if this is the length scale what could be the time scale, what could be the velocity scales, so that we can represent the flow which is kinematic similarity between prototypes and the model.

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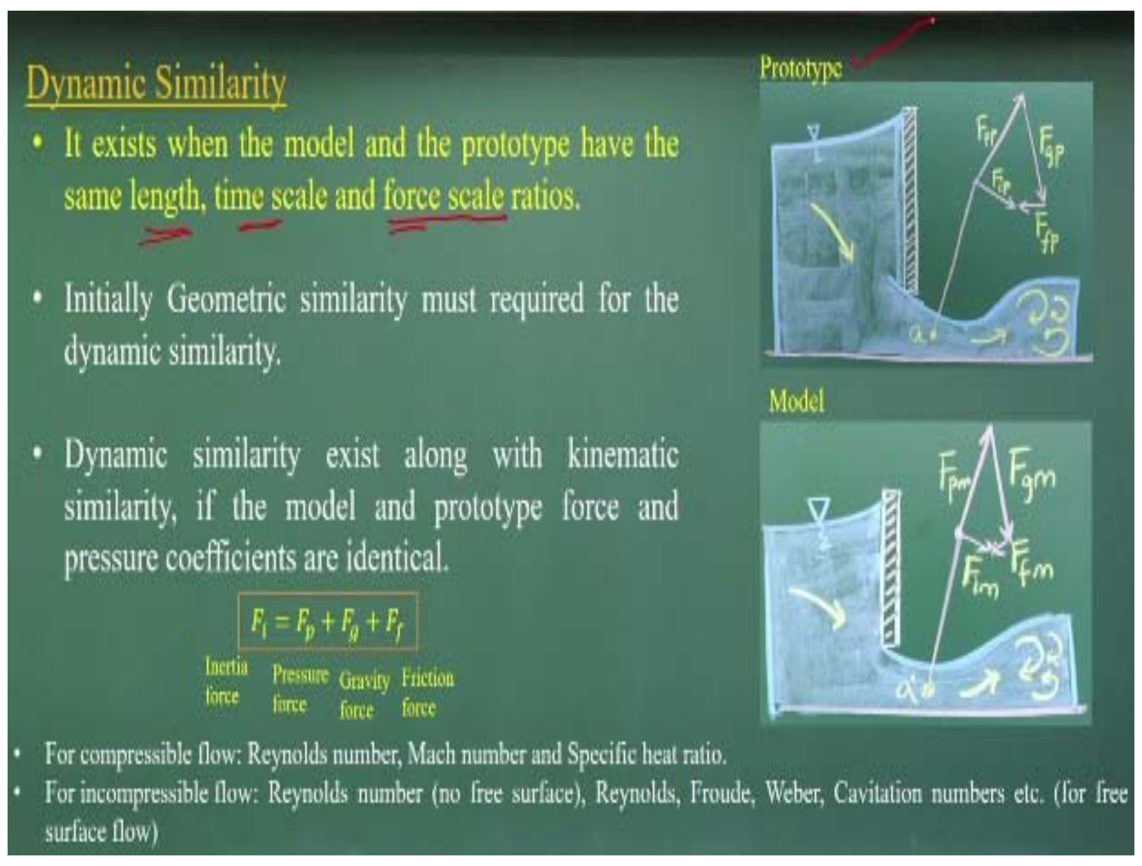

Let us discuss about the dynamic similarity, these similarity adjust when you have the 3 scale ratios, like the length, time and the force. So are the same or model and the prototypes. So the dynamic similarity happens not only this length and the time scale but also the force scale similar ratios will be there, what do you mean by that if you have a let you have a problems like this, you have a prototype, you have a gate.

And flow is passing through gate and having the hydraulic jump here with having a small eddies or vortex formations like this. This is the point where we are looking at what are the force components are there as you know it, it will be the pressure force component, gravity force component, friction force or the viscosity force component and inertia force component and each force has the directions component.

$$F_i = F_p + F_g + F_f$$

If you look at these they should have a balancing all these force, vector balancing components at the prototype level. If prototype and models are at dynamic similarity that means the vector diagrams of this force component of the prototypes and the model should be exactly the same. Force magnitude make the difference but the vector diagrams, vectorical diagrams (()) (30:35) all these component should be exactly same.

So that the force all these component should be exactly same if it is that we can make a ratio between forces of inertia force of the model and the prototype this would be equal. So if you look at these concept when it hold good we call dynamic similarity to achieve the dynamic similarity first you should have geometric similarity as well as the kinematic similarity. If you do not have these 2 similarity, we cannot achieve the dynamic similarity.

So when a dynamic similarity are the same the model prototype force and pressure components are identical that means the ratio look the same. So if you look at this way the force what is acting is here and the if you look at this the diagrams, the force vectorical diagrams of model and prototypes they should have the same. If you exactly know from the vector calculus, we can compute it but will be the ratio between them.

The ratio between them should be equal otherwise the vectorical additions of these exact similar will not be (()) (31:52) you make it the ratios are the same, the directions and the ratios are the same value. Mostly if you look at these for the compressible flow, we use the Reynolds numbers, Mach numbers specific heat ratios but incompressible to without free surface flow conditions we mostly the Reynolds numbers.

But others like numbers also comes, the Reynolds number, Froude number, Weber, Cavitations number what we discuss can be used for free surface flow.

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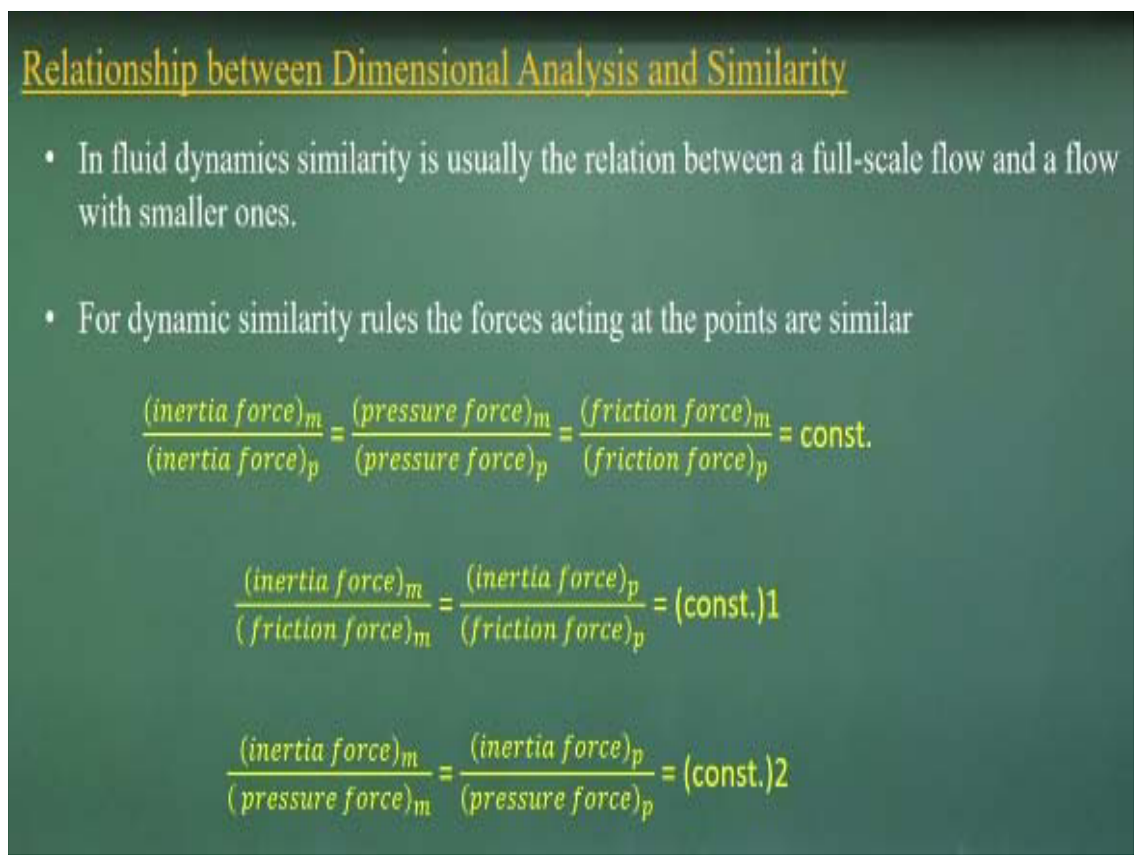

Now let us derive the relationship between the dimensional analysis and the similarity. As I told from the vector diagrams of the force components that the dynamic similarities force acting these points are similar in these case the inertia force of model and the prototype, pressure force of model and prototype, friction force are the viscous flow model prototype should be constant.

Then dynamic similarity it is achieved and that is the case so now we can just reshuffle these equations to get it at the model level and the prototype level we get a one constant. Similar way we write inertia force and the pressure force, inertia force we get another constant. So you are equating as I said it earlier we equate the flow Froude number similarity with in case of free surface flow.

$$\frac{(inertia\ force)_m}{(inertia\ force)_p} = \frac{(pressure\ force)_m}{(pressure\ force)_p} = \frac{(friction\ force)_m}{(friction\ force)_p} = \text{const.}$$

$$\frac{(inertia\ force)_m}{(friction\ force)_m} = \frac{(inertia\ force)_p}{(friction\ force)_p} = (\text{const.})1$$

$$\frac{(inertia\ force)_m}{(pressure\ force)_m} = \frac{(inertia\ force)_p}{(pressure\ force)_p} = (\text{const.})2$$

So similar way we can establish these equations this is the Reynolds number equation that this is what the Euler equations part, what we can get it by equating this part.

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